Mental Calculation progression map – April 2024

Hawes Primary School

> ‘Revisiting mental work at different times during a daily maths lesson or even devoting a whole lesson to it from time to time, helps children to generate confidence in themselves and a feeling that they control calculations rather than the calculation controls them. Opportunities should be made to introduce short periods of mental calculations in other lessons or outside lessons when queuing for some activity. Regular short practise is essential – if you don’t use it you will end up losing it!’

Teaching Children to Calculate Mentally - Department for Education 2010

At Hawes Primary School we believe that children should be introduced to the processes of calculation through practical, oral and mental activities. As children begin to understand the underlying ideas they develop ways of recording to support their thinking and calculation methods, use particular methods that apply to special cases, and learn to interpret and use the signs and symbols involved. Over time children learn how to use models and images, such as empty number lines, to support their mental and informal written methods of calculation. As children’s mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient written methods that can be used more generally. By the end of Year 6 children are equipped with mental and written methods that they understand and can use correctly. When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy. At whatever stage in their learning, and whatever method is being used, it must still be underpinned by a secure and appropriate knowledge of number facts, along with those mental skills that are needed to carry out the process and judge if it was successful.

The overall aim is that when children leave Hawes Primary, they:

• Have a secure knowledge of number facts and a good understanding of the four operations.

• Are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers.

• Make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads.

• Have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally.

The National Curriculum for mathematics aims to ensure that all pupils:

- Become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils have conceptual understanding and are able to recall and apply their knowledge rapidly and accurately to problems.
- Reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language.
- Can solve problems by applying their mathematics to a variety and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Mental Methods of Calculation

Oral and mental work in mathematics is essential, particularly so in calculation. Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts. Later work must ensure that children recognise how the operations relate to one another and how the rules and laws of arithmetic are to be used and applied. Ongoing oral and mental work provides practice and

consolidation of these ideas. It must give children the opportunity to apply what they have learned ‘Revisiting mental work at different times during a daily maths lesson or even devoting a whole lesson to it from time to time, helps children to generate confidence in themselves and a feeling that they control calculations rather than the calculation controls them. Opportunities should be made to introduce short periods of mental calculations in other lessons or outside lessons when queuing for some activity. Regular short practise is essential – if you don’t use it you will end up losing it!’ Teaching Children to Calculate Mentally - Department for Education 2010 4 to particular cases, exemplifying how the rules and laws work, and to general cases where children make decisions and choices for themselves. The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A good knowledge of numbers or a ‘feel’ for numbers is the product of structured practice and repetition. It requires an understanding of number patterns and relationships developed through directed enquiry, use of models and images and the application of acquired number knowledge and skills. Secure mental calculation requires the ability to recall key number facts instantly. In order to ensure that all children at our school achieve their maximum potential in Mathematics in a secure and enabling learning environment, this Policy for Mental Calculation sets out the stages of progression in mental calculation and highlights the strategies to be taught. Throughout the years, children will be introduced to the processes of calculation through lively and fun practical, oral and mental activities, leading on to written methods.

Key points to remember when planning and teaching maths mental calculation:

• Commit regular time to teaching mental calculation strategies.

• Provide practice time with frequent opportunities for children to use one or more facts that they already know to work out more facts.

• Introduce practical approaches and jottings with models and images children can use to carry out calculations as they secure mental strategies.

• Engage children in discussion when they explain their methods and strategies to you and their peers.

Questions for reviewing oral and mental work in mathematics teaching and learning.

• Within each strand, what opportunities are there for children to rehearse and what do they rehearse?

• Are there key facts, other than number facts, that children should recall quickly and accurately?

• How does oral and mental work provide children with the opportunity to refresh and consolidate their previous learning?

• How does oral and mental work help children to refine, reinforce and use, with increasing precision, key aspects of mathematics across all strands?

• What mathematics do children read? Is reading mathematics a feature of each strand and what role does the oral and mental work play?

• How is children’s ability to reason developed by oral and mental work? How is this work planned and organised across all strands?

The Six Rs of Oral and Mental Work

The table below identifies six features of children's mathematical learning that oral and mental work can support. There is a brief description of the learning focus and an outline of possible activities. These are not independent: oral and mental work may address more than one feature of learning and have more than one purpose. What is important is that the activity is purposeful and children understand what they are engaged in and required to learn during the oral and mental activity. The six Rs provide a vocabulary and guide to use when identifying the purposes of oral and mental work; they are not meant to provide a coverage checklist.

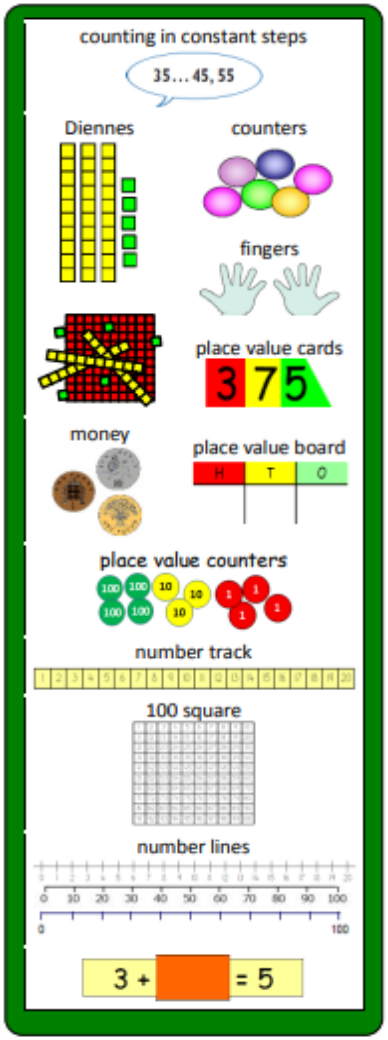
	Learning focus	Possible activities
Rehearse	To practise and consolidate existing skills, usually mental calculation skills, set in a context to involve children in problem solving through the use and application of these skills; use of vocabulary and language of number, properties of shapes or describing and reasoning.	Interpret words such as *more*, *less*, *sum*, *altogether*, *difference*, *subtract*; find missing numbers or missing angles on a straight line; say the number of days in four weeks or the number of 5p coins that make up 35p; describe part-revealed shapes, hidden solids; describe patterns or relationships; explain decisions or why something meets criteria.
Recall	To secure knowledge of facts, usually number facts; build up speed and accuracy; recall quickly names and properties of shapes, units of measure or types of charts, graphs to represent data.	Count on and back in steps of constant size; recite the 6-times table and derive associated division facts; name a shape with five sides or a solid with five flat faces; list properties of cuboids; state units of time and their relationships.
Refresh	To draw on and revisit previous learning; to assess, review and strengthen children's previously acquired knowledge and skills relevant to later learning; return to aspects of mathematics with which the children have had difficulty; draw out key points from learning.	Refresh multiplication facts or properties of shapes and associated vocabulary; find factor pairs for given multiples; return to earlier work on identifying fractional parts of given shapes; locate shapes in a grid as preparation for lesson on coordinates; refer to general cases and identify new cases.
Refine	To sharpen methods and procedures; explain strategies and solutions; extend	Find differences between two two-digit numbers, extend to three-digit numbers to
Read	To use mathematical vocabulary and interpret images, diagrams and symbols correctly; read number sentences and provide equivalents; describe and explain diagrams and features involving scales, tables or graphs; identify shapes from a list of their properties; read and interpret word problems and puzzles; create their own problems and lines of enquiry.	Tell a story using an interactive bar chart, alter the chart for children to retell the story; start with a number sentence (e.g. 2 + 11 = 13) children generate and read equivalent statements for 13; read values on scales with different intervals; read information about a shape and eliminate possible shapes; set number sentences in given contexts; read others' results and offer new questions and ideas for enquiry.
Reason	To use and apply acquired knowledge, skills and understanding; make informed choices and decisions, predict and hypothesise; use deductive reasoning to eliminate or conclude; provide examples that satisfy a condition always, sometimes or never and say why.	Sort shapes into groups and give reasons for selection; discuss why alternative methods of calculation work and when to use them; decide what calculation to do in a problem and explain the choice; deduce a solid from a 2-D picture; use fractions to express proportions; draw conclusions from given statements to solve puzzles.

Progression for Mental Addition

Consider:

- Can I do it in my head using a mental strategy?
- Could I do some jottings?
- Should I use a written method?

Year 1	Year 2	Year 3
• Add and subtract one-digit and two-digit numbers to 20 including zero • Represent and use number bonds and related subtraction facts within 20 • Pupils memorise and reason with number bonds to 10 and 20 in several forms (e.g. 10 + 5 = 15; 15-10 = 5; 15-5=10 • They should realise the effect of adding or subtracting zero.	▪ Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: - A two-digit number and ones, a two-digit number and tens, two two-digit numbers. - Adding three one-digit numbers • Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100.	• Add and subtract numbers mentally, including: - A three-digit number and ones - A three-digit number and tens - A three-digit number and hundreds • Add pairs of 2-digit numbers • Add three small numbers
Mental strategies: • count on in ones; • 1 more than a number; • 10 more than a multiple of 10; • add by counting on from the larger number; • reorder numbers in a calculation; • look for pairs that make 10; • look for doubles and near doubles; • begin to bridge through 10 when adding a one-digit number; • use known facts and place value to add pairs of one-digit numbers; • partition and recombine by breaking units of 6, 7, 8 or 9 into '5 and a bit'; • Add 9 to single-digit numbers by adding 10 then subtracting 1; • use patterns of similar calculations;	**Mental strategies:** • count on in tens or ones; • reorder numbers in a calculation; • add three 1-digit numbers; put the largest number first, using known facts (pairs to 10, doubles); • add by partitioning into tens and ones then recombine; • bridge through a multiple of 10; • use number facts and place value to add pairs of numbers; • add 9, 19, 11 or 21 by rounding and compensating; • use patterns of similar calculations;	**Mental strategies:** • count on in hundreds, tens or ones; • add mentally a 'near multiple of 10'; • add 3 or 4 small numbers; • partition into hundreds, tens and ones and in different ways, then recombine (724 = 700 + 20 + 4) (724 = 600 + 110 + 14); • reorder numbers in a calculation; • bridge through a multiple of 10, then adjust; • use known facts and place value to add; • use patterns of similar calculations; • use the relationship between addition and subtraction;

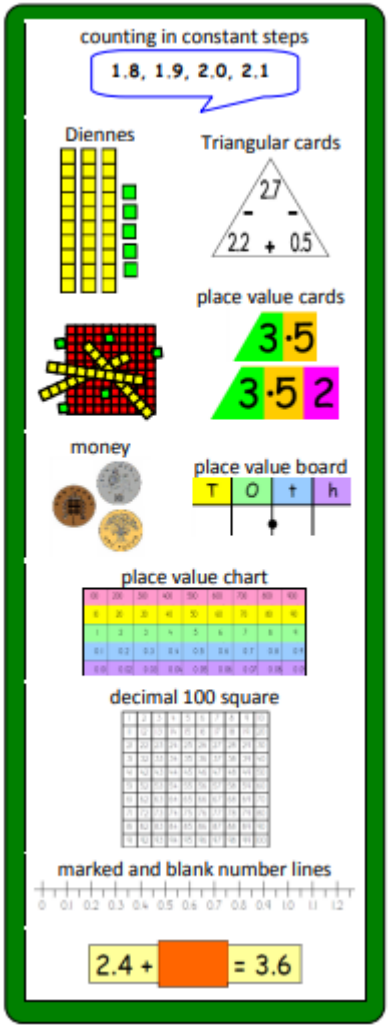


Progression for Mental Addition

Year 4	Year 5	Year 6
• Pupils continue to practise both mental addition and subtraction with increasingly large numbers to aid fluency. • Add a four-digit number and ones, tens hundreds or thousands. • Add a two-digit number to a three-digit tens. • Add any pair of three-digit multiples of ten.	• Add and subtract numbers mentally with increasingly large numbers. • Practise mental calculations with increasingly large numbers. • Mentally add and subtract tenths and one-digit whole numbers and tenths. • Complements to 1 e.g. 0.56 + 0.44 = 1 • Add two numbers with tenths and hundredths.	• Undertake mental calculations with increasingly large numbers and calculations. • Add negative numbers in context. • Add several one-digit whole numbers and tenths. • Add decimals with different number of places. • Add any pair of four-digit multiples of 100. E.g. 4700 + 2800.

Consider:

- Can I do it in my head using a mental strategy?
- Could I do some jottings?
- Should I use a written method?



Mental strategies:

- count on in steps of 1, 10, 100, or 1000;
- reorder numbers in a calculation;
- add 3 or 4 small numbers;
- partition, adding the most significant digit first;
- use known facts and place value to add;
- add the nearest multiple of 10 or 100 then adjust;
- use the relationship between addition and subtraction;

Mental strategies:

- count on in steps of 0.1, 1, 10, 100, or 1000;
- reorder numbers in a calculation;
- partition, adding the most significant digit first;
- use known facts and place value to add;
- add the nearest multiple of 1, 10 or 100 then adjust;
- develop further the relationship between addition and subtraction;

Mental strategies:

- consolidate all strategies from previous years;
- partition, adding the most significant digit first;
- use known facts and place value to add;
- add the nearest multiple of 0.1, 10, 100 or 1000, then adjust;
- continue to use the relationship between addition and subtraction;

Progression for Mental Subtraction

Year 1	Year 2	Year 3
• Subtract a pair of single-digit numbers. E.g. 8 – 3 = 5 • Subtract a single-digit number from a teens number. E.g. 17 – 3 = 14. • Count back in ones.	• Subtraction facts for all numbers up to at least 10. • Subtract any single-digit number from a multiple of 10. E.g. 70 – 4 • Subtract a multiple of 10 from a two-digit number. E.g. 52 – 30 • Subtract a single-digit number from a two-digit number crossing the tens boundary. E.g. 67 – 3, then 62 – 5.	• Subtraction facts for all numbers up to 20, using inverse operations to check. • Subtract groups of small numbers. • Subtract a two-digit number from a multiple of 10. • Subtract two-digit numbers.

Mental strategies:

- count back in ones;
- 1 less than a number;
- 10 less than a multiple of 10;
- take away a small number by counting back;
- find a small difference by counting on *(using equipment);*
- begin to bridge through 10, when subtracting a one-digit number;
- use known number facts and place value to subtract one-digit numbers;
- use patterns of similar calculations;

Mental strategies:

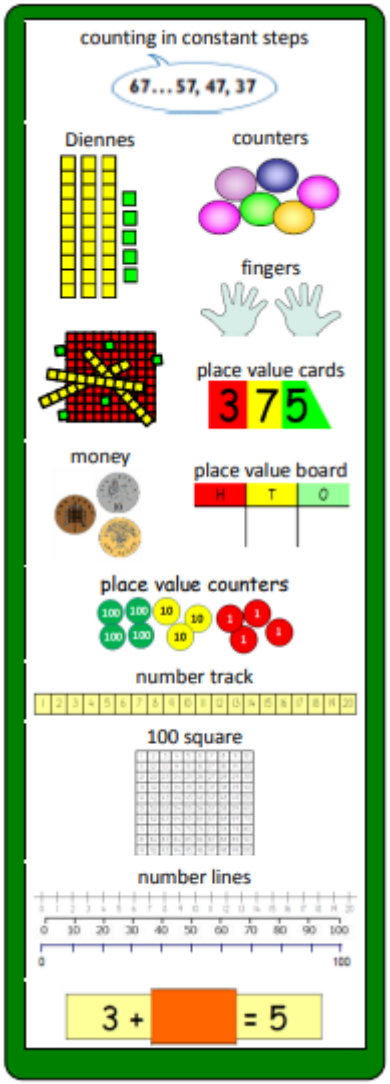
- count back in tens or ones;
- subtract mentally a 'near multiple of 10';
- take away a small number by counting back:
- find a small difference by counting up from the smaller to the larger number (on a number line);
- bridge through a multiple of 10, then adjust;
- use knowledge of number facts and place value to subtract pairs of numbers;
- subtract by partitioning second number and subtracting tens then ones;
- use patterns of similar calculations;

Mental strategies:

- count back in hundreds, tens or ones;
- subtract mentally a 'near multiple of 10';
- find a small difference by counting up from the smaller to the larger number (on a number line);
- bridge through a multiple of 10, then adjust;
- use knowledge of number facts and place value to subtract pairs of numbers;
- subtract a 2-digit number by partitioning it subtracting its tens then ones;
- use patterns of similar calculations;
- use the relationship between addition and subtraction;

Consider:

- Can I do it in my head using a mental strategy?
- Could I do some jottings?
- Should I use a written method?



Progression for Mental Subtraction

Year 4	Year 5	Year 6
• Subtract any pair of two-digit numbers, including crossing the tens and 100 boundaries. • Subtract a near multiple of 10. • Subtract two-digit or three-digit multiples of 10. E.g. 120 – 40. • Count back in hundreds, tens or ones.	• Subtract a pair of two-digit numbers of three-digit multiples of 10. • Subtract a near multiple of 10 or 100 to any two-digit or three-digit number. • Find the difference between near multiples of 100. • Subtract any pair of decimal fractions each with units and tenths. E.g. 7.3 – 5.8 • Count back in hundreds, tens, ones or tenths.	• Subtraction facts for multiples of 10 to 1000 and decimals numbers with one decimal place. • Subtract pairs of decimals with units, tenths and hundredths. • Subtract a decimal with units and tenths, that is nearly a whole number. • Count back in hundreds, tens, ones, tenths or hundredths. • Count back in minutes or hours, bridging through 60.
Mental strategies: • count back in steps of 1, 10, 100, or 1000; • use known facts and place value to subtract ; • find a difference by counting up through the next multiple of 10, 100 or 1000; • subtract the nearest multiple of 10 or 100 then adjust; • use the relationship between addition and subtraction;	**Mental strategies:** • count back in steps of 0.1, 1, 10, 100, or 1000; • use known facts and place value to subtract ; • find a difference by counting up through the next multiple of 10, 100 or 1000; • subtract the nearest multiple of 1, 10 or 100 then adjust; • develop further the relationship between addition and subtraction;	**Mental strategies:** • consolidate all strategies from previous years; • use known facts and place value to subtract; • find a difference by counting up through the next multiple of 10, 100 or 1000; • subtract the nearest multiple of 0.1, 10, 100 or 1000, then adjust; • continue to use the relationship between addition and subtraction;

Consider:

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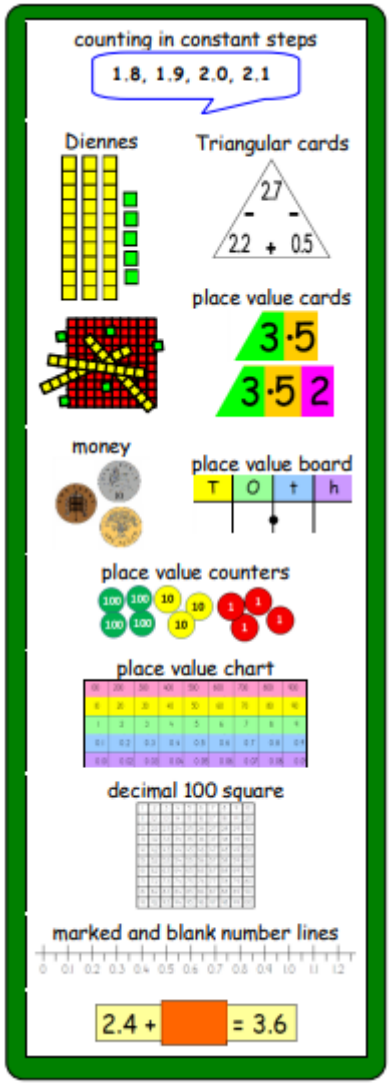

Progression for Mental Multiplication

Year 1

- Give children opportunities to count on in 2s, 5s, and 10's.
- Doubles of all numbers up to 10.
- Use practical problem-solving activities involving counting sets or groups.
- Make connections between arrays and number patterns.
- Odd and even numbers to 20.

Mental strategies:

- counting in twos, fives and tens;
- repeated addition;
- links to doubling;
- use arrays;

Year 2

- Recall and use multiplication facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- Doubles of all numbers up to 20.
- Solve problems using arrays, repeated addition, mental methods and multiplication facts.
- Odd and even numbers to 100.

Mental strategies:

- counting in twos, fives and tens;
- repeated addition;
- use arrays;
- use known facts and place value to multiply by 2, 5 or 10;
- links to doubling;
- reorder a calculation, knowing multiplication can be done in any order (**commutative**);

Year 3

- Recall and use multiplication facts for the 3, 4- and 8-times tables.
- Multiply a 'teens' number by 2, 3, 4, 5 or 8. E.g. 16 x 3
- Multiply a one-digit by a multiple of 10.
- Multiply a two-digit by a one-digit number. E.g. 32 x 3
- Doubles to 50.
- Multiply 3 numbers within known tables. E.g. 3 x 2 x 8

Mental strategies:

- counting in 2s, 5s, 10s, 3s, 4s and 8s;
- repeated addition;
- use known facts and place value to multiply by 2, 3, 4, 5 8 or 10;
- use doubles to link x2, x4 and x8 tables;
- reorder a calculation using **commutativity;**
- use the rule of **associativity;**
- **scaling** up using known facts;
- use the relationship between multiplication and division;

Consider:

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- Could I do some jottings?
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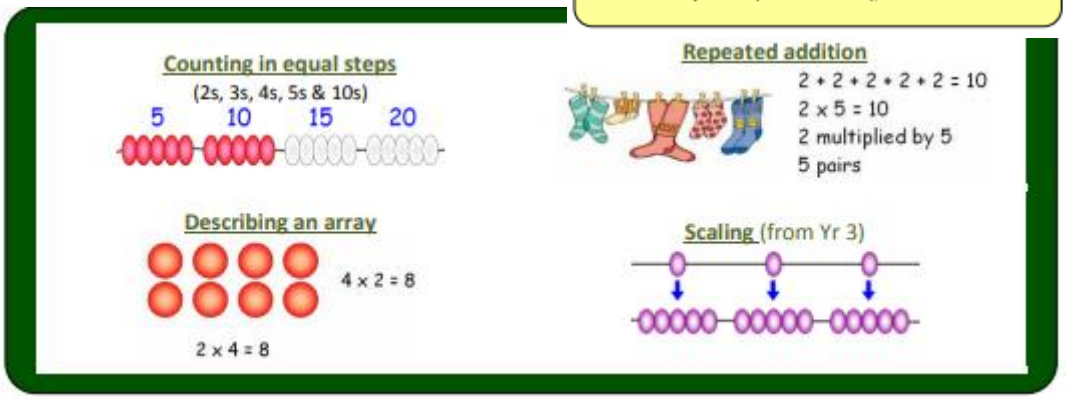

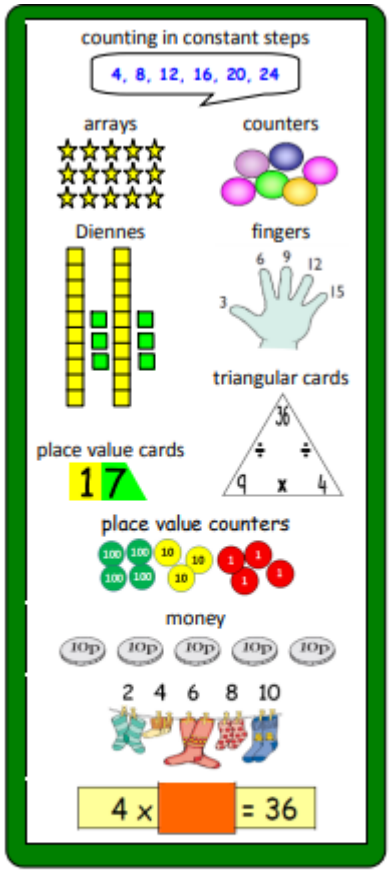

Commutative law

- 2 x 3 = 3 x 2

Associative law

- (2 x 3) x 4 = 2 x (3 x 4)

Progression for Mental Multiplication

Year 4	Year 5	Year 6
• Double any two-digit number. • Double any multiple of 10 or 100. • Multiply numbers to 1000 by 10 and then 100. • Multiply numbers to 20 by a single-digit. • Multiply 3 numbers. E.g. 8 x 7 x 5.	• Multiply numbers mentally drawing upon known facts. • Multiply whole numbers and those involving decimals by 10, 100 and 1000. • Solve problems involving multiplication, using their knowledge of factors and multiples, squares and cubes. • Multiply a two-digit by a one-digit number. • Multiply a multiple of 10 by a multiple of ten. • Double any multiple of 5 up to 500.	• Multiply one-digit numbers with up to two decimal places by whole numbers. • Multiply by 10, 100 or 1000 where the answers are up to 3dp. • Multiply decimals by whole numbers. • Multiply a multiple of 10 by a multiple of 100. • Squares to 12 x 12. • Multiply a unit and tenths number by a one-digit number. • Double decimals less than 1.

Consider:

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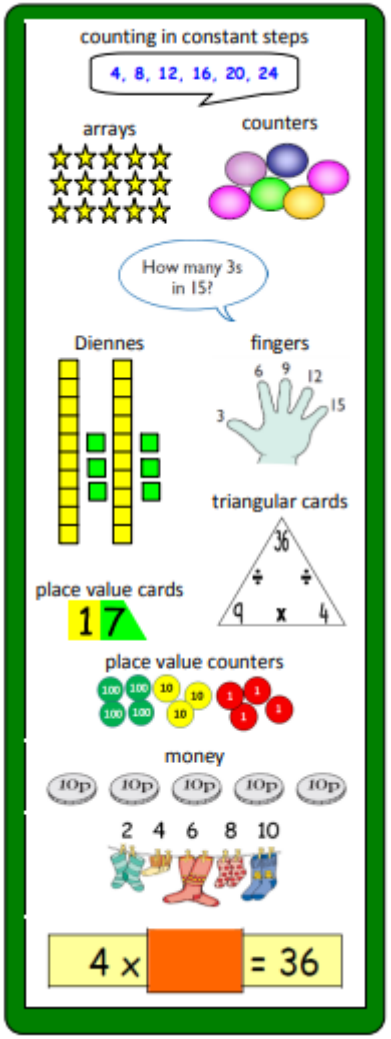
- Should I use a written method?



Mental strategies:

- counting in 6, 7, 9, 25 and 1000;
- use commutativity and tables to multiply;
- use partitioning and **Distributive Law to multiply**;
- use **factor pairs** and the **Associative Law** to multiply;
- use known facts and place value to multiply;
- use related facts to multiply;
- scaling up using known facts;

Mental strategies:

- counting in steps of powers of 10;
- use commutativity and tables to multiply;
- use partitioning and **Distributive Law** to multiply;
- use **factor pairs** and the **Associative Law** to multiply;
- use known facts and place value to multiply;
- use related facts to multiply;
- scaling up using known facts;
- use the relationship between multiplication and division;
- recognise and use square and cube numbers;

Mental strategies:

- use commutativity and tables to multiply;
- use partitioning and the **Distributive Law to multiply;**
- use **factor pairs** and the **Associative Law** to multiply;
- use known facts and place value to multiply;
- use related facts to multiply;
- scaling up using known facts;
- use the relationship between multiplication and division;

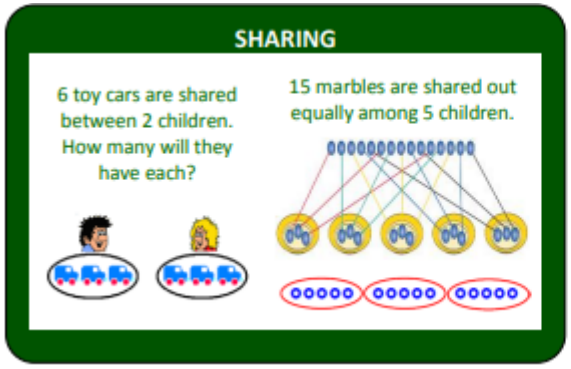
Progression for Mental Division

Year 1

- Count back to 0 in 2s, 5s and 10s.
- Solve simple one-step problems involving division, by using practical objects and arrays with support.
- Halves or corresponding doubles to 10.

Mental strategies:

- counting in twos, fives and tens;
- links to halving;
- use arrays;

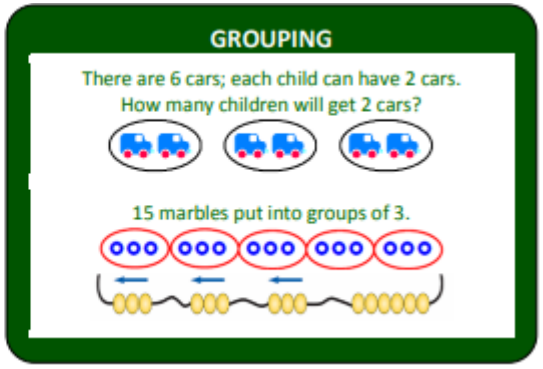


Year 2

- Recall and use division facts for 2, 5- and 10-times tables, including recognising odd and even numbers.
- Halves of corresponding doubles to 20.
- Divide a two-digit number by 2, 5 or 10 to give a 'teens' answer. E.g. 34 ÷ 2 = 17
- Find half of even numbers to 40.
- Halve any multiple of 10 up to 100. E.g. halve 70.

Mental strategies:

- counting in twos, fives and tens;
- link to arrays;
- use known facts and place value to divide;
- partition in different ways to divide;
- links to halving;

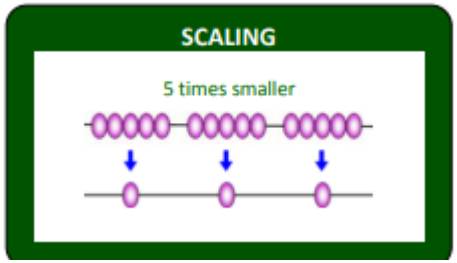


Year 3

- Recall and use division facts for the 3, 4- and 8-times tables.
- Dive a number by 3, 4 or 8 to give a 'teens' answer. E.g. 42 ÷ 3.
- Divide a ten number by a one-digit or tens number.
- Divide a two or three-digit number by 3, 4, or 8.
- Halves of corresponding numbers to 50.
- Halves any multiple of 10 up to 200.

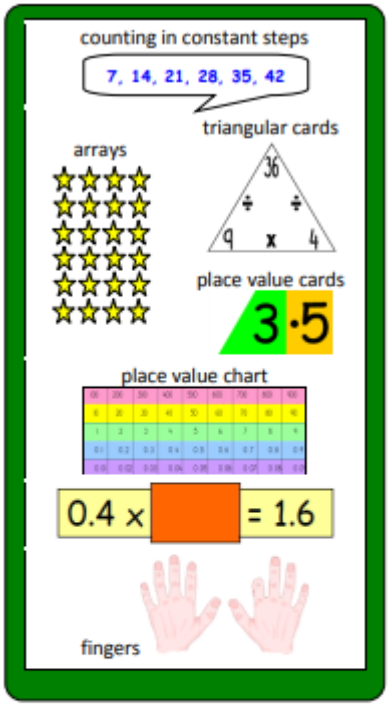
Mental strategies:

- counting in 2s, 5s, 10s, 3s, 4s and 8s;
- use known facts and place value to divide by 2, 3, 4, 5 8 or 10;
- partition in different ways to divide;
- use halving to link ÷8, ÷4 and ÷2 tables;
- **scaling** down using known facts;
- use the relationship between multiplication and division;



Consider:

- Can I do it in my head using a mental strategy?
- Could I do some jottings?
- Should I use a written method?



Factor pairs

- 2 x 3 = 6, 6 has a factor pair of 2 and 3

Distributive law

- 39 x 7 = 30 x 7 + 9 x 7

Associative law

- 14 x 12 = (2 x 7) x 12 = 2 x (7 x 12)

Progression for Mental Division

Year 4	Year 5	Year 6
• Recall division facts up to 12 x 12. • Use place value, known and derived facts to divide mentally. • Practise mental methods and extend this to three-digit numbers to derive facts. E.g. 400 ÷ 2 = 200 can derived by 2 x 2 = 4. • Halve any even number to 200. • Identify the remainder when dividing by 2, 5 or 10. • Divide a number to give a 'teens' answer.	• Divide numbers mentally drawing upon known facts. • Divide whole numbers and those involving decimals by 10, 100 or 1000. • Divide a three-digit number by a one-digit. • Halves of corresponding doubles of any multiple of 5 up to 500. • Divide a multiple of 10 by a single-digit number • Divide by 4 or 8 by repeated halving. • Use knowledge of division facts to find factor pairs.	• Pupils are introduced to the division of decimal numbers by one-digit whole numbers. • Divide numbers by 10, 100 or 1000, where the answers are up to 3dp. • Divide multiples of 100 by a multiple of 10 or 100 (whole number answers). • Divide two-digit decimals. • Find 10% of whole numbers and quantities. • Halves of corresponding doubles of ones and tenths and decimals less than 1 (2dp). •

Mental strategies:

- counting in 6, 7, 9, 25 and 1000;
- use partitioning and the **Distributive Law to divide**;
- use known facts and place value to divide;
- use related facts to divide;
- use **factor pairs** to divide;
- scaling down using known facts;
- use the relationship between multiplication and division;
- **include calculations with remainders;**

Mental strategies:

- counting in steps of powers of 10
- use partitioning and the **Distributive law to divide**;
- use known facts and place value to divide;
- use related facts to divide;
- use **factor pairs to** divide;
- scaling down using known facts;
- use the relationship between multiplication and division;

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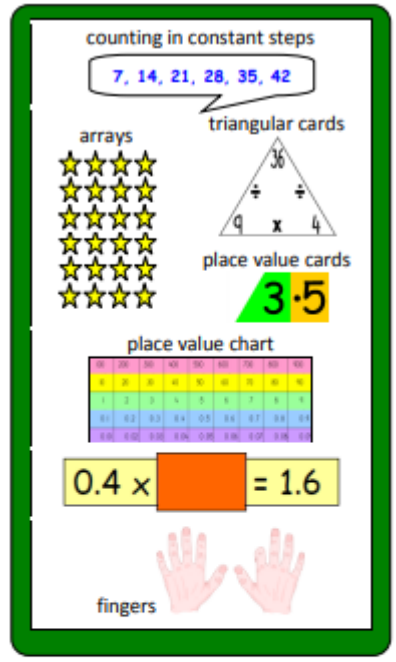
Consider:

- Can I do it in my head using a mental strategy?
- Could I do some jottings?
- Should I use a written method?



Factor pairs

- 15 = 3 x 5, 15 has a factor pair of 3 and 5
- 600 ÷ 15 = 600 ÷ 3 ÷ 5

Distributive law

- 98 ÷ 7 = (70 ÷ 7) + (28 ÷ 7)

Link to finding fractions of amounts and quantities